A Generalized Numbering System for Esperanto In Bases Other Than Ten

An essay about the connection between languages, numbers, Esperanto, and a practical proposal for a small expansion of the international language.

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Note: If you are here just for the number-naming proposal, skip right to PART THREE. Everything else is just a long essay, and some personal thoughts and observations about languages, numbers and Esperanto. Don't worry, you won't miss it if you just want to see the end result and not the long idea behind it. I do recommend reading the rest if you are curious enough, since it was interesting to write it, to me at least.

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https://archive.org/details/esperanto-and-number-naming-in-other-bases

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A Proposal for a general way to generate proper naming for all number-bases in Esperanto.

1 Introduction

Language is arguably one of the most important areas of human life, just after the bare minimum survivalist areas like eating, and having shelter from the harsh conditions of the capricious weather of our little planet. A person who cannot speak, or have a proper form of communication with others will be forever doomed to be isolated from his fellow humans. Language is what separates us from animals, not just for the complexity of the sounds we make, since there are other animals that have the potential to create even more complex sounds than us with just our limited anatomical larynxes. However, it is not the mere sounds that make language, but the way we can express complex ideas, and generate understanding between one another with an amalgamation of sounds systematically, purposefully, and carefully placed one after the other, in order to create the right words that convey just the precise meaning that we want to express and make ourselves be understood by others, and even sometimes, to understand ourselves.

Furthermore, each society that has ever attained any significant civilizational development, has created ways to convey those words, not just by sounds, but also through written symbols that are associated, in one way or another, to those words, facilitating the communication between people that are not physically close to one another, since the development of the technology that now allows us to have instantaneous communication through the world is a novelty created just during the last decades, and even one could argue that they were developed BECAUSE of the development of the written word.

However, as useful as language is, it is also one of the greatest barriers of today. In an ever more connected world, we find ourselves divided between the people of one nation to the next, because each nation, isolated from the other, usually by more than just the political separation of each nation, but by physical barriers, which in ancient times were near impossible to cross, such as mountains, cliffs, canyons, seas, rivers, and oceans. These separations between each nation, and the inevitable evolution of any local language, which, as time goes by, always tends to get more and more distorted when compared to the older versions than the previous generations used, even if the changes are so subtle that they are almost imperceptible between two consecutive generations. The addition of these changes, in time, all decanted into the generation of completely different languages between each region and even nation of Earth.

Another of the necessary areas of human life, one that is responsible for most of the development of modern civilization (if not all of it), has been mathematics and all the magnificent operations that we can use numbers for. From simple transactions in our local market, or distribute in equal parts between siblings a box of candies, up until the complex calculations made by computers that allow us to send satellites into space, which gave us telecommunications, and other wonders like GPS, portable phones, and credit cards. Numbers and mathematics are indeed one of the most important areas of human life.

However, just like what happened with languages, and the way that each culture and nation developed different spoken words, so too differ our written symbolism for expressing words. And the way in which we express numbers, both by sounds and by symbols, is no different in this aspect. Except that numbers have an added layer of complexity, since the numbers themselves are not something tangible, but they represent an idea; in its most primitive form, numbers represent the “how many” or the “how much” of anything that we can measure or count. But since we can always count “the next one” to each number with no limit, numbers themselves cannot have a limit. This leads to the problem of how to “express” a number, both in written and spoken form, since there seems to be no limitation to the amount of words and symbols that would be needed for them.

How did we solve this problem? How to name each of the infinite amount of numbers? The answer: By recursive additions and iterations of the symbols and the words we use. We choose a limited number of words, and a limited number of symbols, and group them together, in a systematic order, and we have a systematic way to name those numbers, which also is repetitive and “agglutinative”. However, the way that this recursive naming and representation has been used is wildly different for each culture. We will see some clarifying examples of this concept later in Part One.

Aside from the obvious need to be able to translate names properly between languages, it is also clear that many areas of mathematics and science could benefit greatly from having the proper language to express numbers in other bases. For computer scientists, for example, could be beneficial to have a naming system for base-two (binary), or any of its superior potency-base, namely base-four, base-eight, or even base-sixteen (hexadecimal). Unfortunately, most of these bases have no proper naming system in no language, because no language was ever developed with them.

Another base system that is a favorite among mathematicians is base-twelve (dozenal), because of several properties of number twelve that makes it, arguably, a better base than base-ten (decimal), even if it would be impractical to even attempt a wide scale switch to dozenal. Nonetheless, it is worth noting that people do talk about this subject, and the need to be able to have a proper naming system comes with it.

However, in most languages, all these bases have no proper naming system, since they never required it. And also, the translation of numbers from one language to another may not even be compatible if said languages do not use a similar naming system for numbers, more on this later in PART ONE.

Now, over a hundredth years ago, there was a man who dared to dream of a world where every person on the planet could communicate with each other in a common language, but not by forced imposition, and certainly not as a replacement to their original languages, since there are many valuable things that are created because of the differences between each language, both the spoken words, and the symbols used to write them. That man was named Ludwik Lejzer Zamenhof. What he envisioned is a world where everybody would learn a simple and easy second language, what is called an auxiliary language, that could be used as a means of understanding between people. He believed that this would help to bring about peace among the nations. He presented his original language proposal in 1887, in a book titled “Dr. Esperanto's International Language” under the pseudonym “Dr. Esperanto”, which, in his proposed language, the word “Esperanto” means “the one that is hopeful”.

In his proposal, Zamenhof included a piece of it to a number naming system, with a base-ten (decimal) proposal, which is the most important number system of today. And in this aspect, it makes a far better job at expressing numbers than most, if not all, the rest of natural languages today. However, because of Esperanto’s simplicity, numbers written ONLY in base-ten can be properly expressed. Fortunately, Zamenhof himself did not wanted to be a “dictator” of the language, and soon after he made his proposal, he “liberated” the language for the public to use and modify however they wanted.

Here is where this proposal enters, not to replace the original number system of Esperanto. I do not propose this to start any “great reform” neither to the language Esperanto, nor to the way we write and name numbers on a daily basis, but only to “expand” the possibilities, and hopefully bring about a better understanding between people, from all backgrounds, whenever a proper translation of numbers is needed. Hopefully this will be useful to somebody, or perhaps to nobody, but it is made solely with the intention of helping someone interested in it.

2 PART 1: Languages and numbers

2.1 The way that number names and their symbols are connected.

Since the dawn of civilizations, we have required numbers for many different purposes. And we have developed many ways to express them, both in spoken form, and in "symbolic" one. For example, the romans used symbolic system that used a kind of "base-five/base-ten" symbolic nature:

I=One/Unit; V=Five; X=Ten; L=Fifty; C=Hundred; M=thousand.

However, their system included sums and subtractions to represent a single number:

"XLVII" is the roman number "Forty-Seven", although the symbols used actually mean "XL= Ten units less than fifty" plus "VII = Five and two more units".

This kind of expressions made even the simplest arithmetical operations terribly complex. Fortunately, in modern times, practically the entire world uses a much more efficient way to write numbers, symbolically at least, which is the "positional-digits system", by using a set of symbols that represent a limited amount of numbers, and depending on their location in the symbol, they actually represent a multiple of a potency of another number called the "base", which in most countries around the world is selected to be "ten".

Let's check the English language and its modern writing system for numbers. The symbols used in this system are:

0=zero/nothing; 1=one; 2=two; 3=three; 4=four; 5=five; 6=six; 7=seven; 8=eight; 9=nine.

The rest of numbers can be represented by positioning the same symbols one after the other, with the left-most number as the one with the highest value, and the right-most one as the one with the least.

This is how we can write the symbol "6539" which is just a set of four characters placed one after the other, and we can name it as "Six Thousand Five Hundredth and thirty-nine" despite the fact that it's a very large number. All because we have given a name to each character, and another name to each position. Like this, we know that the first character (6) is called "Six", and because of its location, it is in the position of the "thousands", similarly the "five" (5) is in the "hundredths" position, the "three" (3) is in the "tens" position, and finally the "nine" (9) is in the units position.

However, notice one small detail. When naming the numbers, we didn't say "three tens", but "thirTY", because the English language developed the particle-word "TY", which can be translated to something like "times ten":

20=TwenTY; 30=ThirTY; 40=FourTY; … 90=NineTY.

And in order to name other numbers that include these "tens-placing", just name the next number: "35"="Thirty-Five"; "67"="Sixty-Seven".

However, this does not work with numbers lower than twenty:

10= ten; 11=eleven; 12=twelve; 13=Thirteen; 14=fourteen; … ; 19=nineteen

We notice two facts here:

(1) there is a unique name for each number up until twelve.

(2) There is the particle-word “TEEN” added at the end of each of the other numbers until nineteen. So, we can interpret this particle word as “plus ten”.

The first fact could indicate that English was at some point influenced by a base-twelve number naming system. And the second could indicate that it was influenced by a “base-twenty” number naming system at some point. The English language kept the names, even if the systems were later replaced by a base-ten symbolic representation.

Anybody that has gone through the basic courses of primary school in any English-speaking country understands these basic principles of how “numbers” work.

However, rarely, if ever, is it signaled that the words “tens”, “hundredths”, and “thousands” don’t mean “the second, third, and fourth location to the left” respectively, but they mean something even more precise: “thousands”, actually means “ten*ten*ten”, “hundredths” means “ten*ten”, and the ending “TY” in “forty” actually means “times ten”. Notice how the number “ten” keeps appearing and multiplying itself over and over depending on the “positional-word”. This is what the “base” of the number system is: The number that is elevated to a potency depending on the position that a digit is placed on.

But what if, instead of the number “ten” we would have used another number for the base. Why not “eight”? Why not “twelve”? Why not “Seventy-Three”? In fact, many cultures through history have used different bases to represent their numbers, and therefore given different names to their numbers. A clear example of this can be found in English itself, with the base twelve, which uses the words “dozen” (twelve), “Gross” (twelve* twelve) and “Great-Gross” (twelve* twelve*twelve) to represent the second, third, and fourth positional numbers respectively. In this case, the previous number, represented by “6539” in base “ten” is now represented by: “394B” in base “twelve”. Notice that, although it’s the same number, its symbolic representation has nothing to do with the first symbolism, and now the base-ten name “Six Thousand Five Hundred and Thirty-Nine” is inadequate, since none of the words used in base ten adequately represent the base-twelve symbolism. Therefore, we must also change its name accordingly, and now it is called “three great-grosses, nine grosses, four dozens, and eleven.” This illustrates the problem of changing the names of numbers when there is a different number-base to consider.

2.2 The usefulness of other bases (mathematical reasons):

2.2.1 Computers work in binary: bases two, and sixteen

The need for an adequate naming system for numbers becomes even more obvious with other base changes that have no adequate naming system in English. Base-two (binary), or some of its superior bases like base-sixteen (hexadecimal), are widely needed in areas such as computing, but have no adequate number-naming system, and usually resort back to the naming system of base-ten in one way or another.

Having a proper naming system for numbers in other bases could be useful to even facilitate the understanding of how computer machines work, since what a computer actually does is just processing a series of “ones” and “zeroes” really fast, since that is what a “computer character” actually is, it’s just a symbol associated with a number represented with ones and zeroes. For example, the character “A” (upper case A), is represented by the binary number “1000001”, in what is called the “Ascii” symbol table. Similarly, when calculating large sums of numbers, what a computer does internally is just transforming the number into binary and then just adding and subtracting ones and zeroes.

However, base-ten and base-two generate extremely different ways of expressing the same numbers. For example, this number in binary: “1’100’100” is the decimal number “hundred”, but it’s practically impossible to give a proper name to it in binary, at least in the English language. An alternative to this writing system for treating computer numbers is the use of base-sixteen, because sixteen is just a power of two ($16 = 2^4$), and under this base, the number “hundred” is represented by the symbol “64”, again completely disconnected the name from the symbols written, and no simple way to connect them back.

This illustrates the need for an adequate naming system for other bases. But let’s further add to the discussion with one last historical detail, and an error (small by comparison to other ones) that was committed during the French Revolution.

2.2.2 Base-twelve, a historical review, and a comparison with base-ten.

During the French Revolution, there was an attempt to reform the way that measurements were made. Since there was an enormous number of different units for measuring different quantities (still active and alive known today as the imperial measuring system), and there was no easy way to remember them all or easily transform them from one unit to another. As a result, the International System of Units (SI) was created, which was a great standardization of the unit-system of measurements. And it simply uses multiples and submultiples of “tens” for each important measurement unit and its own multiple bases and submultiple bases. I will not enter in detail about this, but for further reading you can read this summary:

https://physics.info/system-international/summary.shtml/

However, before the selection of “parts of tens” used to divide and multiply each measurement unit, there was a small debate about which base-number to use. And the debate was reduced to selecting either base-ten, or base-twelve. The former was selected. And today it is considered by some to have been a mistake. The reason for this is because base-twelve has significant advantages over base-ten.

To begin with, the multiplication tables would be much more “repetitive” and easier to remember than base-ten. The downside is that there would have been two more tables (for eleven and twelve) to learn, but both would have been relatively simple regardless. You can see a good review of this here:

https://www.youtube.com/watch?v=y_QBDrBlbds

Also, the “unit fractions” would be easier to represent in base twelve:

Fraction	Base-ten	Base-twelve
1/2	0.5	0.6
1/3	0.333333…	0.4
1/4	0.25	0.3
1/5	0.2	0.(2497)(2497)(2497)…
1/6	0.166666…	0.2
1/7	0.(142857)(142857)…	0.(186a35)(186a35)…
1/8	0.125	0.16
1/9	0.1111…	0.14

Notice that of all these fractions, base-twelve performs a worse job than base-ten only in the 1/5 fraction, and the others it is either equally good, or better, since there are almost no recurring numbers, except for 1/5 and 1/7, while base-ten has recurring terms in 1/3, 1/6, 1/7, and 1/9.

2.3 Numerical bases and the disconnection with the naming of words and their symbolical representation, three examples:

2.3.1 An analysis of French and base-ten

For those that have ever wanted to learn French, or that already speak it, you would probably have noticed a small detail in the language when it comes to numbers. The French numbers have a similar way of naming numbers as English for all the numbers from one until sixty-nine. However, once the number Seventy is reached, the naming system becomes confusing to say the least. To see a full list of all the strangeness of the system, read this page:

https://blog.lingodeer.com/french-numbers-counting/#French_Numbers_70_to_79

But only to have a small flavor of the system, we will see some of the names of the numbers here:

Number	French name	Literal translation
70	Soixante-dix	Sixty-ten
71	Soixante-et-onze	Sixty and eleven
79	Soixante-dix-neuf	Sixty-ten-nine
80	Quatre-vingts	Four-twenties
90	Quatre-vingt-dix	Four-twenty-ten
99	Quatre-vingt-dix-neuf	Four-twenty-ten-nine

As it can be seen, starting from seventy and onwards, there is complete disconnection between the words spoken and the numbers written, since nowhere in the symbol “70” there is a “sixty” nor a separated “ten”. And nowhere in the symbol “80” is there neither a single “four”, nor a “twenty”. There must be a historical reason for the French language to have developed like this, and it is most likely the influence of some ancient base-twenty numeral system that was used in France, but that was later replaced by the modern “base-ten” symbol system, but was left stuck with the original base-twenty names in its language.

2.3.2 The development of the Kaktovik-Inupiaq numerals

A similar problem, and an unexpected solution, appeared just some years ago for a completely different language and the people speaking it. In western Alaska there is a small group of natives, called the Inuits, and whose main native language is called "Inupiaq", although it could be more of a "collective" name for several languages and dialects of the region.

However, what is important for this conversation is that their language uses a base-twenty type of numbering system, and within the language they have a sub-basis-five, something similar to what the romans used (5=V and 10=X). However, because of the continuous use of the base-ten numerals and naming system, the Inupiaq children began to lose interest in the numbering system of their native language, and because of the lack of background to properly name base-ten numbers with their own language, they faired relatively worse in arithmetic than their English-speaking peers.

However, during the 1990s, a group of Inupiaq students developed a numeral system that gave a symbol to each number from zero to nineteen. These numerals are now known as the Kaktovik-Inupiaq or just Kaktovik numerals. Here they are in their full form:

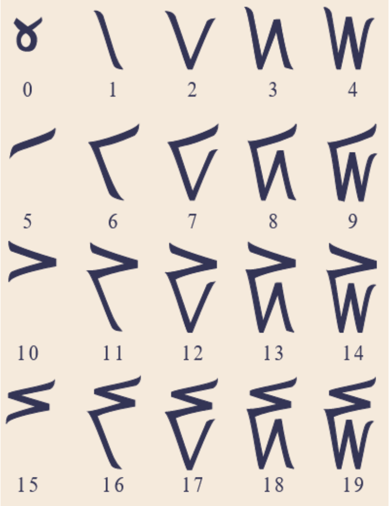

Image source: https://en.wikipedia.org/wiki/File:Kaktovik_digit_table.svg (Wikimedia Commons)

Notice that the numerals are grouped in "fives", with their top part being identical for each row (except for the first one), which indicates the "sub-base-five" of the system, and this is also reflected in the spoken Inupiaq language. Reportedly, these numerals became widespread among the Inupiaq people to be used. Their featural characteristics made them easy to learn and remember, and the immediate connection they have between the written and spoken form of the numerals enabled the Inupiaq children to learn arithmetic with much more ease, and the revival of the linguistic base-twenty of the Inupiaq language became generalized in the region.

You can see more information about this here:

https://en.wikipedia.org/wiki/Kaktovik_numerals

Also, the Youtube channel “Artifexian” made a couple of videos about this subject:

https://www.youtube.com/watch?v=EyS6FfczH0Q

https://www.youtube.com/watch?v=Pas7A1mZbdU

This also illustrates how important it is to have a direct link between the numerical symbols we write and the words we use to express them.

2.4 Historical Bases of Civilizations.

If we look at the development of most civilizations through the world, most of them has developed a numbering system based on five, ten, or even twenty. The reason for this might be because we humans have a great reason to use those particular numbers: they are part of our bodies. We have five fingers on each hand and each foot, amounting to a total of ten fingers in our hands, and twenty fingers overall. So, it is only natural that the first societies developed a way of counting by comparing them to the number of extremely obvious body parts that we all have.

There are, however, some exceptions. Many languages from Papua and New Guinea have much simpler number bases, among them there are some using a form of binary, ternary and even quaternary (https://www.jstor.org/stable/41187615). The Komnzo language, for example, has a built-in base-six numerical system (https://en.wikipedia.org/wiki/K%C3%B3mnzo_language#Numeral_system)

While the number system of the Nimbia language, a dialect of the Gwandara language from Nigeria, uses a base-twelve system (http://www.sf.airnet.ne.jp/~ts/language/number/nimbia.html) (https://en.wikipedia.org/wiki/Gwandara_language).

So, although rare, there are languages that uses counting systems that are incompatible with the base-ten counting system.

2.5 Base sixty and its connection to twelve, or why there are sixty seconds to a minute, sixty minutes to an hour, but twenty-four hours to a day.

As a last point that I would like to touch is that there has been, historically, other number systems that managed to keep some of its influences even today. Babylonians had a base-sixty numbering system with a sub-base-ten (https://mathshistory.st-andrews.ac.uk/HistTopics/Babylonian_numerals/), even if they lacked a symbol for “zero”. They were among the firsts to divide the day in smaller parts, and because of their counting system, it made sense that they would want to divide it into pieces that would coincide with them, so that is why there is sixty seconds to a minute and sixty minutes to an hour. However dividing the day into sixty parts as well would have been excessive, so the next best number to divide a day was twelve, simply because it is easier to select a smaller chunk than that, for example you can take its half (six hours), third (four hours), its fourth (three hours), its sixth (two hours), and its twelfth (one hour), while using base-ten divisions you can only take its half, its fifth, or its tenth. So they divided the day and the night, both in twelve parts, and there you have it, the origin of the twenty-four hours to a day. Also due to the fact that 60=12*5, it means that there was some usefulness to the sub-base-twelve already, so the language associated with a base-twelve would have also came in handy, even for the ancient Babylonians at their time.

3 PART 2: Esperanto: The International Language

3.1 Esperanto's flexibility: Not even its creator dared to chain it.

A long time ago, at the end of the 19th century to be precise, Ludwik Lejzer Zamenhof, an ophthalmologist by profession, became fascinated with the idea of making a language that would become the "second language" of the world. In other words, he wanted to make an auxiliary language, easy to learn for as many as possible, so that people could use it as a means of communication with one another, even if they came from completely different backgrounds.

His objective was not to replace the world's languages with his own creation, but simply to aid the world in creating a culturally neutral, simple, and effective language that could be used for expressing as many subjects as they are possible in the human languages all over the world.

With this in mind, in 1887, he launched a first book called "Dr. Esperanto's International Language" (https://genekeyes.com/Dr_Esperanto.html), under the pseudonym "Doktoro Esperanto", or "The doctor who is hopeful", and later the name "Esperanto" became associated with the language itself. Today that book is also known as "la unua libro", or "the first book" written about Esperanto as a language. Later on, Zamenhof published a second book called "Dua Libro de l' Lingvo Internacia", or "the second book of the international language", in which he further expanded a bit his ideas for the language (http://esperanto.davidgsimpson.com/librejo/dualibro.html).

However, Esperanto was not the first (nor the last) attempt at trying to create an auxiliary language. Another previous attempt was made with "Volapük" in 1880 by Johann Martin Schleyer, a german catholic priest. However, Schleyer made a mistake: he tried to control the development of Volapük and retain his proprietary rights (https://en.wikipedia.org/wiki/Volap%C3%BCk). This, as it should be no surprise to nobody, led to the majority of the initial supporters of the language to get away from it and seek refuge in other auxiliary-language-projects.

When Zamenhof developed "Esperanto", he did not make the same mistake as Schleyer, because once he published his first and second book, he himself signals that he was "liberating" the language to the public. In the closing statement of the "Dua Libro", Zamenhof states this:

> "This booklet is the last word I speak in the role of author. From this day on, the future of the international language is no more in my hands than in the hands of any other friend of the sacred idea. We must now all work equally, each according to his strengths. Each of you can do now as much for our cause as me, and many of you can do much more than I, for I am without capital, and of my time, occupied with work for daily bread, I can sacrifice to the beloved matter only a very small part. I did everything I could for the cause, and if every true friend of the international language brings to it even one hundredth of the moral and material

> sacrifices that I have brought to it over twelve years until today, then the cause will go great and reach the destination in the shortest time. Let's work and hope!”

As can be seen, Zamenhof made with Esperanto what Schleyer failed to do with Volapük: giving the public the opportunity to use, change, and expand the language however they wanted. Whether if he learnt from the mistake of Schleyer or if it was just his own idea, it is not stated anywhere, but the end result is the same: Esperanto expanded rapidly, while Volapük died out.

Even today Esperanto is the most widely spoken international language, even when it was persecuted by both, Hitler’s regime and Stalin’s during the second world war. We must also thank the dawn of the internet for that, since now there is a real possibility to speak to all the Esperanto speakers of the world and use as many resources available to us to learn and expand Esperanto today much more than ever before.

3.2 Esperanto as an international language, and its numbering system.

As it has been repeatedly stated, Esperanto was born with the sole purpose of becoming the “second language” of everybody. However, one detail that Zamenhof included implicitly in the language was its base-ten numbering system, and because of Esperanto’s simplicity, he never developed any language to express numbers in no other base that wasn’t base-ten.

This, as it has been made abundantly clear from the discussion of the previous part, is not necessarily enough to express all the ways that humans can speak about numbers and mathematics, not only because of other languages that don’t use base-ten for their numbers (like the Inupiaq people), but also because people that use base-ten languages may need to speak about numbers in other systems (like computer scientists that need to express numbers in binary more efficiently).

Just as an illustration of how bad is the “official” way of expressing numbers in Esperanto when they are NOT in base ten, I am going to make a comparison with English of the same number in base-ten and base-twelve.

The number Thirty-Five: in base-ten, this number is expressed in the symbol “35”, and its name in Esperanto is “TriDek Kvin”, literally “ThreeTen Five”, which perfectly connects the symbols written with the words used to express it. Now, if we express the same number in base-twelve, the symbol used to represent it becomes “2B”, and now the name “Thirty-Five” has become completely disconnected between the written symbols and the spoken name, since now there is no “three” in the tens position, or a “five” in the units, but there is a “two” in the “twelves” position and an “eleven” in the units. In English this can be fixed by taking advantage of the already existing language-words associated with it, and we can call it “two dozens and eleven”. While trying to do the same in Esperanto, it would be something like “Du DekDuoj kaj DekUnu” translated as “Two TenTwos and TenOne”, which resorts back to the “base-ten” number naming, and it becomes very confusing, very fast.

This finally leads us to the very core of this entire essay: the proposed system for expanding the number-naming of Esperanto into other bases other than ten.

4 PART 3: The Proposed System.

4.1 Base ten in Esperanto: the basis for all other bases.

As stated before, Esperanto was developed exclusively for base-ten naming. And as such, any other number naming system should be modified as little as possible as to keep as much of the original words from Esperanto as part of the new number naming system (at least in my personal appreciation). However, it must also make it easy enough as to be able to link the written symbols with the spoken words for it. So, in order to expand the Esperanto language into other bases, we must first behold the current state of Esperanto with regard to its first and only official base: base-ten.

Symbol	Esperanto name
0	Nul
1	Unu
2	Du
3	Tri
4	Kvar
5	Kvin
6	Ses
7	Sep
8	Ok
9	Naŭ
10	Dek
100	Cent
1'000	Mil

The last three numbers (Dek, Cent, Mil), also work as the "positional" words, for naming other numbers. The way that Esperanto builds other numbers is by combining the names of the symbols directly with their positional words, like this:

12 = Dek-Duo; 1'506 = Mil KvinCent Ses; 365'462= TriCent SesDek Kvin Mil KvarCent SesDek Du.

Notice that above the "thousands", Esperanto simply repeats the same logic for naming past the tens of thousands and hundreds of thousands, generating a "grouping" of three digits per level, before requiring a new name above it.

There is no official way to describe numbers higher than this, however, historically, Esperanto developed other words derived from two systems of naming higher numbers, and a third one that is the closes to an "official" way of naming, because it simply added two root words, that have a specific meaning, and mixes them with the other number-words from Esperanto itself. Taken from the Esperanto-Wikipedia (https://eo.wikipedia.org/wiki/Vortoj_por_grandegaj_nombroj):

Nombro	Nombrovorto laŭ "Mallonga" Skalo	Nombrovorto laŭ "Longa" Skalo	Preferinda nombrovorto en Esperanto
10^6	Miliono	Miliono	Miliono
10^9	Biliono	Miliardo	Miliardo
10^{12}	Triliono	Biliono	Duiliono
10^{15}	Kvadriliono	Biliardo	Duiliardo
10^{18}	Kvintiliono	Triliono	Triiliono
10^{21}	Sekstiliono	Triliardo	Triiliardo

10^{24}	Septiliono	Kvadriliono	Kvariliono
10^{27}	Oktiliono	Kvadriliardo	Kvariliardo
10^{30}	Noniliono	Kvintiliono	Kviniliono
10^{33}	Deciliono	Kvintiliardo	Kviniliardo
10^{36}	Undeciliono	Sekstiliono	Sesiliono
10^{39}	Duodeciliono	Sekstiliardo	Sesiliardo
10^{42}	Tredeciliono	Septiliono	Sepiliono
10^{45}	Kvatuordeciliono	Septiliardo	Sepiliardo
10^{48}	Kvindeciliono	Oktiliono	Okiliono
10^{51}	Seksdeciliono	Oktiliardo	Okiliardo
10^{54}	Septendeciliono	Noniliono	Naŭiliono
10^{57}	Oktodeciliono	Noniliardo	Naŭiliardo
10^{60}	Novemdeciliono	Deciliono	Dekiliono
10^{63}	Vigintiliono	Deciliardo	Dekiliardo
10^{66}	Unvigintiliono	Undeciliono	Dek alt sesdek ses
10^{69}	Duovigintiliono	Undeciliardo	Dek alt sesdek naŭ

As can be seen, the names from the two center columns are influenced by the latin language in order to name the “zilions”. However, it is the column on the far right that is of interest to us. As can be seen, the words in the last columns can be separated into the amalgamation of two root-words, one representing an Esperanto number-word (except for the first two that use an “m-” instead of “unu-”) and another that is either “-iliono” or “-iliardo”. The meaning of these last two root-words can be expressed with a literal mathematical formula:

$$\mathrm{N} + \mathrm{iliono} = 10^{6*N}$$

$$N + iliardo = 10^{6*\mathrm{N}+3}$$

And finally, if we look at the last two rows, of the far right column, we can see that past the “Dekiliardo”, the preferred name uses the “alt-” (high) word to express a potency of ten: “Dek alt sesdek ses” means “Ten High (elevated to) Sixty-Six”, and “Dek alt sesdek naŭ” means “Ten High (elevated to) Sixty Nine”.

One last detail, Esperanto has another root suffix to express fractional numbers. That root suffix is “-on”, and is used to signal when something is divided into equal parts. For example, “duono” means “a half”, “triono” means “a third”, “dekono” would mean “a tenth”. So, the number “0.5” could be named as “kvin dekonoj” or “five tenths”. In general, this is the naming system for decimals: you name the number written to the right of the decimal point, and the place of its last digit with the “-ono” or “-onoj” suffix attached to it, and both separated with the word “el” in between:

0.25 = “dudek-kvin + “el” + cent-onoj”,

“(25).(3’567) = (dudek kvin), kaj (tri mil kvincent sesdek sep + “el” + dek-mil-onoj)”

With all these lineaments, we can name any number expressed in base-ten to whatever precision we require, both high numbers, and very small ones.

For the rest of bases, I will keep the majority of these concepts, but will leave behind some of the most painfully difficult to deal with.

4.2 Base twelve: the only one with (almost) appropriate words apart from base ten.

Now, although there are no “official” words for use in base twelve, there has been adaptations of words from other languages that do have base-twelve compatible words, that have been absorbed by the Esperanto community along the years. Namely, we have the words:

“Dozeno” for “twelve”: (https://glosbe.com/eo/en/dozeno),

“Groco” for “twelve*twelve” (https://eo.wikipedia.org/wiki/Groco_(nombro)), and

“Maso” for “twelve*twelve*twelve” (https://eo.wikipedia.org/wiki/Maso_(nombro))

However, because the base-ten counting system uses monosyllables to express each name, we must modify these ones a little bit to fit the proper naming system.

The modification for “Dozeno” cannot be “Doz” nor “Do” because there are words already in Esperanto that are associated with them “Dozo” is a “dose” of some medicine for example, and “do” is the equivalent of the English word “so”. As a result, the modification that occurred to me was “doc” that comes from the Spanish word “docena” = dozen.

For the other two words, we just eliminate the final “o” at the end: “Groc” and “Mas”. This is also an accepted rule of Esperanto that can be used with any noun.

Now, the last thing we need is a name for the number “eleven”, and the word “elv” doesn’t have any other associated meaning to it in Esperanto, so this one was selected for this. With these names, the basic system now reads:

Symbol	Esperanto name
0	Nul
1	Unu
2	Du
3	Tri
4	Kvar
5	Kvin
6	Ses
7	Sep
8	Ok
9	Naŭ
A	Dek
B	Elv
10	Doc
100	Groc
1’000	Mas

To further develop the system past this point, it is necessary to have some further rules that will be general to all other bases.

4.3 The rules of the base-naming system

4.3.1 The names of the numbers and digits

Before we start this discussion, it is important to distinguish the “symbol” of a number and the “number” it represents. I will use plain base-ten English language to express the “number” we talk about, but the “symbols” will represent different numbers depending on the base that is being talked about. Like this, the symbol “10” may represent ten, twelve, two, three, etc. depending on the “base” that we are talking about. This applies to any symbol that is comprised of more than one digit, because the “positional system” is dependant on the base used. So, when in doubt, remember that the written numbers “101”, “101” and “101” may represent completely different things depending on which base they are expressed.

For lower bases than ten, we will use the same names that base-ten gives, until the first highest number with a one followed by zeros that can be expressed with it. To illustrate this, let’s take a look at binary and ternary:

For base-two: the names of all the relevant symbols are:

Symbol	Esperanto name
0	Nul
1	Unu
10	Du
11	Tri
100	Kvar
101	Kvin
110	Ses
111	Sep
1’000	Ok

However, numbers above this one (1’000=“ok”) will not maintain their original names, because the next potency of “10” (two) is “10’000” which is actually sixteen, and this one doesn’t have a one-syllable word in Esperanto. Interestingly enough, binary is also the only one that doesn’t need names for further grouping numbers, since “10”=du, “100”=kvar, and “1’000”=ok (look at the next section to see what I mean with this).

For base-three: the same relevant table would be:

Symbol	Esperanto name
0	Nul
1	Unu
2	Du
10	Tri
11	Kvar
12	Kvin
20	Ses
21	Sep
22	Ok
100	Naŭ

This one also has names for the symbol “10” and “100”, but not for “1’000”, since this one is the number “twenty-seven”, so it is going to need a new name for it.

For bases above three: the original base-ten numbers can only be used for single digits alone and the symbol “10”, since, beginning with **base four**, the symbol “100” which is sixteen in base four, lies outside the “ten” original numbers, and for higher bases this simply gets even higher, so base four will be the “template” for all the other bases that require a new definition of the symbols “100” and “1’000”

Symbol	Esperanto name
0	Nul
1	Unu
2	Du
3	Tri
10	Kvar

For bases above ten: It is necessary to have a new root-word for each new digit that must be included. It is preferable that they have some connection to some language, and that they are also monosyllables, but the most important detail is that their names do no collide with any basic Esperanto-definition.

With some effort I managed to find words that kind-of accomplish this, up until the number **twenty**. Making all the bases under twenty available for use. It is important to use these new words ONLY if the system needs over ten digits, and only until the max name of the base. Any other use of them would be too different from the “Esperanto” original numbers.

Decimal number	Single Digit Symbol	Esperanto name	Name Origin
0	0	Nul	
1	1	Unu	
2	2	Du	
3	3	Tri	
4	4	Kvar	
5	5	Kvin	
6	6	Ses	
7	7	Sep	
8	8	Ok	
9	9	Naŭ	
10	A	Dek	
11	B	Elv	English-eleven
12	C	Doc	Spanish-dozen
13	D	Draj	German-three
14	E	Vjer	German-four
15	F	Fajv	English-five
16	G	Liŭ	Chinese-six
17	H	Siv	German-seven
18	I	Ejg	English-eight
19	J	Nŭev	Spanish-nine
20	K	Vejm	Spanish-twenty

4.3.2 The Symbols “10”, “100”, “1’000” and their names

As seen in the section about base-ten, the positional naming of numbers works marvellously, the only thing required is to have proper names for each position. In base ten, the positional names “dek”, “cent” and “mil” worked to represent these positions. Also, as seen in the previous section, binary already gives names to these three symbols, and ternary to the first two, while the bases higher than three require new names for both the positional “100” and “1’000”, and the name of “10” is the same as the base of the system.

The simplest way to give new names to these positional numbers would be to use modifications of the base-ten originals “cent” and “mil”. So using the particle “centa-“ and “mila-“ attached to the esperanto-name of the base would be a viable solution, even if a bit messy.

To clarify this, we will once more revisit bases two, three and four:

For base two: This one already has a name for all the basic symbols, since “1’000” is just eight (ok).

10’110 = du ok ses.

111’111 sep ok sep.

1’011 ok tri.

Notice that the only “positional” word is “ok”, while all the other “three digit groups” have a single-word name.

For base three: this one requires a new name just for the symbol “1’000” and it would be “mila-“+”tri” so “milatri”

And with the names of all the other symbols we can now represent the next numbers:

2’210 = du milatri dunaŭ tri

12’111 = kvin milatri naŭ kvar

212’221=dunaŭ kvin milatri dunaŭ sep

Notice that the only “positional” words are “milatri” and “naŭ”. We are naming the numbers, not just in a spell-it-out digit-by-digit, but rather in a “use the least amount of positionals available”.

For base four: This one requires names for both “100” and “1’000” and they are “centakvar” and “milakvar” respectively. Now we can name the next numbers:

332’103 = tri-centakvar trikvar du milakvar centakvar tri

23’221 = dukvar tri milakvar ducentakvar dukvar unu.

1’010 = milakvar kvar

Notice that this is the first base that actually uses all the positional words “kvar” for “10”, “centakvar” for “100”, and “milakvar” for “1’000”.

All other bases work similarly to base 4, except for base twelve, which was actually given separate names for each positional symbol.

For base twelve: as stated before, “10” is “doc”, “100” is “groc” and “1’000” is “mas”.

BA9’876 = elvgroc dekdoc naŭ mas okgroc sepdoc ses.

10’010 = doc mas doc.

1’45B = mas kvargroc kvindoc elv.

This one does use all the positional word-particles assigned to it.

4.3.3 Higher numbers: multiples of “1’000”

Naming numbers over the “1’000” positional number is another problem. A possible solution is to use either a modification of the root-words “-iliono” and “-iliardo” as in base ten, or to invent new words to replace them.

However, there is another way. In English, for base-twelve there is a way to name numbers beyond the “gross” (100), and it is to simply add the word “great” before it as many times as needed. Like this 1’000 is a great-gross, 10’000 is a great-great-gross, 100’000 is a great-great-great-gross and so on. However, this gets really tiresome after just the first few iterations, and if we ever need really big numbers, this gets, from annoying, to unmanageable. However, this could be an inspiration to a system that takes advantage of this “recursive” naming.

On the other hand, in the original Esperanto base-ten naming system there already is a simple, yet effective way to name numbers that are really big: simply say the power of “ten” that they are elevated to. Like this, the base-ten number 10^{69} is now named “Dek alt SesDek Naŭ”. Notice that the word “alt(a)”, which means “high”, is used before naming the power to which “10” must be elevated to.

So, taking advantage from this, we can use a similar system. Except that now we can use it, not over the symbol “10”, but over the symbol “1’000” to get its own powers named. Nevertheless, to keep some semblance to the original base-ten naming system, the number representing the power should be placed before the word “alta”, and ending with the name for the symbol “1’000” for the base. Also, for the sake of internal coherence, the number-name of the power must be consistently named within the same base. This will be clear with a couple of examples.

For base two:

We can finally name the number “hundred” in binary:

1’100’100 = duAlta-ok kvar ok kvar

$1'000^{1'100}$ = ok-kvarAlta-ok

Notice that we are using this huge number to show the naming of the power is also base-two.

For base three:

21’202’011 = sep duAlta-milatri (dunaŭ du) milatri kvar.

$1'000^{201'000}$ = dunaŭ-unu-milatriAlta-milatri.

Again, this huge number is used only to illustrate that the power is also named in base-three.

For base Four:

123’000’000’000 = (centakvar dukvar tri) triAlta-milatri.

$1'000^{32'021}$ = trikvar-du-milakvar-dukvar-unuAlta-milatri.

For base Twelve:

A92’0B0’000’000 = (DekGroc NaŭDoc Du) triAlta-mas ElvDoc du-alta-milatri.

$1'000^{A0'0B7}$= DekDoc-Mas-ElvDoc-SepAlta-milatri.

Note: in a previous incarnation of a naming system (for base twelve but that is besides the point), I had used the root word “obla” which means “multiple” instead of “alta”, but that was before I knew of the use of “alt” to indicate a potency, so I decided to be as complicit with conventions as possible.

4.3.4 Fraction naming: just name the number and add “-ono” at the ending position.

So far, we have named only whole numbers, but not any non-whole number. The usual name for non-whole numbers is “decimals”, but that name would be inadequate for any non-base-ten system, so we are sticking to “non-whole number” as a descriptor.

To fully name a number, you need to express its “whole” part, and its “fractional” part separately, with the word “punkto” in between, and simply give the ending position for the fractional number with the particle-word “-ono” attached to it and the word “el” to separate the fractional number from its positional descriptor. This will be made clear with the next examples:

For base-two:

(101).(01’011’010) = (Kvin) punkto (du-alta-ok tri ok du) “el” kvar-du-alta-ok-onoj.

Notice that the number to the right of the point could be expressed as 1’011’010/100’000’000, and it is the denominator of this fraction that marks the “ending position” of the fractional section of the number.

For base-three:

0.(00201) = (dunaŭ unu) “el” naŭ-milatri-onoj

Again, notice that the fractional number can be expressed as 201/100’000, and the denominator is what gives its positional name.

For base-four:

(2’000).(12’005) = (du milakvar) punkto (kvar du milakvar kvin) el centakvar-milakvar-onoj.

Again, the fractional part can be expressed as 12’005/100’000, and the denominator gives its positional name.

For base-twelve:

(A9).(0B) = (DekDoc Naŭ) punkto (elv) el groc-onoj.

4.4 Summary of several non-base-ten systems: two, three, four (template for all the others), twelve, and twenty.

Base two:

Symbol	Esperanto name
0	Nul
1	Unu
10	Du
11	Tri
100	Kvar
101	Kvin
110	Ses
111	Sep
1’000	Ok

Number	Name	Fraction Number	Fraction name
1;	unu		
10;	du	**0;1**	du-ono
100;	kvar	**0;01**	kvar-ono
1,000;	ok	**0;001**	ok-ono
10,000;	du-ok	**0;000,1**	du-ok-ono
100,000;	kvar-ok	**0;000,01**	kvar-ok-ono
1,000,000;	dualta-ok	**0;000,001**	dualta-ok -ono
10,000,000;	du-dualta-ok	**0;000,000,1**	du-dualta-ok-ono
100,000,000;	kvar-dualta-ok	**0;000,000,01**	kvar-dualta-ok-ono
1,000,000,000;	trialta-ok	**0;000,000,001**	trialta-ok-ono

Base-three:

Symbol	Esperanto name
0	Nul
1	Unu
2	Du
10	Tri
11	Kvar
12	Kvin
20	Ses
21	Sep
22	Ok
100	Naŭ

Number	Name	Fraction Number	Fraction name
1;	unu		
10;	tri	**0;1**	tri-ono
100;	naŭ	**0;01**	naŭ-ono
1,000;	milatri	**0;001**	milatri-ono
10,000;	tri-milatri	**0;000,1**	tri-milatri-ono
100,000;	naŭ-milatri	**0;000,01**	naŭ-milatri-ono
1,000,000;	dualta-milatri	**0;000,001**	dualta-milatri-ono
10,000,000;	tri-dualta-milatri	**0;000,000,1**	tri-dualta-milatri-ono
100,000,000;	naŭ-dualta-milatri	**0;000,000,01**	naŭ-dualta-milatri-ono
1,000,000,000;	trialta-milatri	**0;000,000,001**	trialta-milatri-ono

Base Four (template for all other bases (except twelve and ten), just add more digits and change the base name used for the “10” (kvar), “100” (centakvar) and “1’000” (milakvar) symbols):

Symbol	Esperanto name
0	Nul
1	Unu
2	Du
3	Tri
10	Kvar

Number	Name	Fraction Number	Fraction name
1;	unu		
10;	kvar	**0;1**	kvar-ono
100;	centakvar	**0;01**	centakvar-ono
1,000;	milakvar	**0;001**	milakvar-ono
10,000;	kvar-milakvar	**0;000,1**	kvar-milakvar-ono
100,000;	centakvar-milakvar	**0;000,01**	centakvar-milakvar-ono
1,000,000;	dualta-milakvar	**0;000,001**	dualta-milakvar-ono
10,000,000;	kvar-dualta-milakvar	**0;000,000,1**	kvar-dualta-milakvar-ono
100,000,000;	centakvar-dualta-milakvar	**0;000,000,01**	centakvar-dualta-milakvar-ono
1,000,000,000;	trialta-milakvar	**0;000,000,001**	trialta-milakvar-ono

Base Twelve:

Symbol	Esperanto name
0	Nul
1	Unu
2	Du
3	Tri
4	Kvar
5	Kvin
6	Ses
7	Sep
8	Ok
9	Naŭ
A	Dek
B	Elv
10	Doc

Number	Name	Fraction Number	Fraction name
1;	unu		
10;	doc	**0;1**	doc-ono
100;	groc	**0;01**	groc-ono
1,000;	mas	**0;001**	mas-ono
10,000;	doc-mas	**0;000,1**	doc-mas-ono
100,000;	groc-mas	**0;000,01**	groc-mas-ono
1,000,000;	dualta-mas	**0;000,001**	dualta-mas-ono
10,000,000;	doc-dualta-mas	**0;000,000,1**	doc-dualta-mas-ono
100,000,000;	groc-dualta-mas	**0;000,000,01**	groc-dualta-mas-ono
1,000,000,000;	trialta-mas	**0;000,000,001**	trialta-mas-ono

Finally, don’t forget the table with the names of **digits for superior bases**:

Decimal number	Single Digit Symbol	Esperanto name
0	0	Nul
1	1	Unu
2	2	Du
3	3	Tri
4	4	Kvar
5	5	Kvin
6	6	Ses
7	7	Sep
8	8	Ok
9	9	Naŭ
10	A	Dek
11	B	Elv
12	C	Doc
13	D	Draj
14	E	Vjer
15	F	Fajv
16	G	Liŭ
17	H	Siv
18	I	Ejg
19	J	Nŭev
20	K	Vejm

Wow. So much effort for such a useless subject, is it not? Well, it’s been fun. I hope this piece of my personal insanity is useful for somebody. If not, well, at least I enjoyed making it.